

problem: $\boxed{\left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2} = x}$

$$\left(\frac{x^2-1}{x}\right)^{1/2} + \left(\frac{x-1}{x}\right)^{1/2} = x = \frac{\sqrt{x^2-1}}{\sqrt{x}} + \frac{\sqrt{x-1}}{\sqrt{x}}$$

$$x\sqrt{x} = \sqrt{x^2-1} + \sqrt{x-1}$$

$$\left[x\sqrt{x} - \sqrt{x-1} = \sqrt{x^2-1}\right] \Rightarrow \left[(x\sqrt{x} - \sqrt{x-1})^2 = x^2-1\right]$$

$$x^3 - 2x\sqrt{x}\sqrt{x-1} + (x-1) = x^2-1$$

$$\cancel{x}(x^2 - 2\sqrt{x}\sqrt{x-1}) = x^2 - x = \cancel{x}(x-1)$$

cancel

$$x^2 - 2\sqrt{x}\sqrt{x-1} = (x-1)$$

$$x^2 - x + 1 = 2\sqrt{x}\sqrt{x-1}$$

$$(x^2 - x) + 1 = 2\sqrt{x^2 - x}$$

notice $(\sqrt{x}\sqrt{x-1})^2 = x(x-1) = (x^2 - x)$

so $\sqrt{x}\sqrt{x-1} = \sqrt{x^2 - x}$

$$1 = 2\sqrt{x^2 - x} - (x^2 - x) \leftarrow \text{quadratic we are after}$$

$$Q = -(x^2 - x) + 2\sqrt{x^2 - x} - 1$$

$$-b^2 + 2b - 1 \text{ with } b = \sqrt{x^2 - x}$$

so root at $b=1$ $\left[1 = \sqrt{x^2 - x}\right] \text{ so } \left[1 = x^2 - x\right]$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{matrix} a = -1 \\ b = 2 \\ c = -1 \end{matrix}$$

$$\frac{-2 \pm \sqrt{4 - 4(-1)(-1)}}{2(-1)} = \frac{-2 \pm 2}{-2} = 1$$

This gives another quadratic $a = 1$

$$Q = x^2 - x - 1$$

$$\begin{matrix} b = -1 \\ c = -1 \end{matrix}$$

$$\frac{1 \pm \sqrt{1 - 4(-1)}}{2} \Rightarrow \frac{1 \pm \sqrt{5}}{2}$$

checking solution and determining (+) or (-) for second quadratic.

$$\left(\frac{1 + \sqrt{5}}{2}\right)^2 \stackrel{(+)}{=} \frac{1 + 2\sqrt{5} + 5}{4} = \frac{6 + 2\sqrt{5}}{4} = \frac{3 + \sqrt{5}}{2}$$

$$\stackrel{(-)}{=} \frac{1 - 2\sqrt{5} + 5}{4} = \frac{6 - 2\sqrt{5}}{4} = \frac{3 - \sqrt{5}}{2}$$

using (-) $\rightarrow \left[\left(\frac{3 - \sqrt{5}}{2}\right) - \left(\frac{1 - \sqrt{5}}{2}\right) - 1 = 0\right] \Rightarrow 3 - \sqrt{5} - 1 + \sqrt{5} - 2 = 0$
 $\boxed{3 - 1 - 2 = 0}$ Valid soln for #2

$$\left(\frac{1-\sqrt{5}}{2} - \frac{2}{1-\sqrt{5}}\right)^{1/2} + \left(1 - \frac{2}{1-\sqrt{5}}\right)^{1/2} = \times$$

$$(1-\sqrt{5})^2 \quad 1-2\sqrt{5}+5 = (6-2\sqrt{5}) =$$

$$\frac{(6-2\sqrt{5})-4}{2(1-\sqrt{5})} + \left(\frac{1-\sqrt{5}-2}{1-\sqrt{5}}\right)^{1/2} =$$

$$\frac{2-2\sqrt{5}}{2(1-\sqrt{5})}$$

$$\left(\frac{-1-\sqrt{5}}{1-\sqrt{5}}\right)^{1/2} =$$

$$\left(\frac{1-\sqrt{5}}{1-\sqrt{5}}\right)$$

$$1$$

$$+ \left(\frac{-1-\sqrt{5}}{1-\sqrt{5}}\right)^{1/2} = \frac{1-\sqrt{5}}{2}$$

$$\left(\frac{-1-\sqrt{5}}{1-\sqrt{5}}\right) = \left(\frac{1-\sqrt{5}}{2} - 1\right)^2$$

$$\frac{-1-\sqrt{5}}{1-\sqrt{5}} = \frac{6-2\sqrt{5}}{4} - \frac{2(1-\sqrt{5})}{2} + 1$$

$$\frac{6}{4} - \frac{2}{4}\sqrt{5} - 1 + \sqrt{5} + 1$$

$$= \frac{3}{2} - \frac{1}{2}\sqrt{5} + \sqrt{5}$$

$$(-1-\sqrt{5}) = \left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right)(1-\sqrt{5})$$

$$\frac{3}{2} - \frac{3\sqrt{5}}{2} + \frac{\sqrt{5}}{2} - \frac{5}{2}$$

$$\frac{3}{2} - \frac{\sqrt{5}}{2} - \frac{5}{2}$$

$$-\frac{2}{2} - \sqrt{5}$$

$$(1-\sqrt{5}) = (-1-\sqrt{5})$$