Without actually expanding solve: $(12x-1)(6x-1)(4x-1)(3x-1)=5$

sub $a=(12x-1)$ $\Downarrow$ $(12x-1)\left(\frac{2(6x-1)}{2}\right)\left(\frac{3(4x-1)}{3}\right)\left(\frac{4(3x-1)}{4}\right)=5$

$$a\left(\frac{a-1}{2}\right)\left(\frac{a-2}{3}\right)\left(\frac{a-3}{4}\right)=5$$

$$\boxed{a(a-1)(a-2)(a-3)=120}$$

Ricks solution

Rick looked at the equation saying a must be a multiple of 5, he also saw it as needing to be less than 10 but I don't get that.

$(5)(4)(3)(2)=120$

$20 \quad 6 \quad = 120$

$\boxed{120=120}$

It works, but I want to see if I can get there without the Jump / Insight step.

Just like the power series for computing $\sum N^3$

$a(a-1)(a-2)(a-3)=120$

Nest the multiplications!

$(a^2-3a)(a^2-3a+2)=120$

$(a^2-3a)^2+2(a^2-3a)-120=\phi$

$B=(a^2-3a)$

$B^2+2B-120=\phi$

$a=1$ $\quad b=2$ $\quad c=-120$ $\quad \frac{-2\pm\sqrt{4+4(120)}}{2}$

$\sqrt{4+4(120)}=\sqrt{4(1+120)}=2\sqrt{121}=2(11)=22$

$\Downarrow$

$\frac{-2\pm 22}{2}\Rightarrow\left\{-\frac{24}{2},+\frac{20}{2}\right\}$

$\Rightarrow\{-12,10\}$

$B=\{-12,10\}$

Path A

Path B

$-12=a^2-3a$

$\phi=a^2-3a+12$

$a'=1$
$b'=-3$
$c'=12$

$\frac{3\pm\sqrt{9-48}}{2}$

$\downarrow$

Imaginary so not soln path here. since we have a real valued solution!

$10=a^2-3a$

$\phi=a^2-3a-10$

$a'=1$
$b'=-3$
$c'=-10$

$\frac{3\pm\sqrt{9-4(-10)}}{2}=\frac{3\pm\sqrt{49}}{2}$

$a=\frac{3\pm 7}{2}=\{-2,5\}$

$\Downarrow$

Both solutions work

Checking solutions

$(-12)^2+2(-12)=120$

$144 \quad -24 \quad =120$

$10^2+2(10)=120$

Both solutions check

which we can show since

$\left[a(a-1)(a-2)(a-3)=120\right]\Big|_{\{-2,5\}}\Rightarrow$ $(-2)(-3)(-4)(-5)=120$, $(5)(4)(3)(2)=120$

No we can expand for $\boxed{a=(12x-1)}$

$-2=12x-1$

$-1=12x$

$\boxed{-1/12=x}$

$5=12x-1$

$6=12x$

$\frac{6}{12}=\boxed{x=\frac{1}{2}}$